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REVIEW

Mathematical modelling of some geodynamical problems

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ABSTRACT

Tectonic earthquakes occur when the stress of Earth's crust exceeds the instant shear strength. Determining the stress state of a particular area using the equations of mechanics requires knowledge of the boundary conditions, which in most cases seem impossible. This paper proposes the determination of horizontal stresses for the Central Asian territory using the well-known geodynamic hypothesis, according to which the deformation of the Earth's crust in Central Asia is due to the interaction of the Indian, Arabian, and Eurasian plates. The unknown boundary conditions are reconstructed by solving the inverse problem of elasticity. Some known empirical stresses are used to verify the problem. The solution of the elastic problem makes it possible to set the problem of tectonic creep movements using the Stokes equations. The model is verified by means of horizontal velocity and rotation fields constructed from GPS data. The creep model makes it possible to determine the vertical velocities of the Earth's crust and supplements the GPS data. The constructed stress state model is used to calculate the variation in the earth's crust stresses due to earthquakes. A double dipole without a moment is taken as the source mechanism of earthquakes. The boundary element method (BEM) is used for the numerical realization of the model.

KEYWORDS

Geodynamics; Tectonic stresses; Elasticity; Creep; Mathematical model

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Introduction

The study of modern tectonic stresses at different scale levels is becoming increasingly important in the Earth sciences. The determination of these fields is very promising for new poses, not only purely scientific but also for prospecting, prognostic, geo-ecological, and other applied problems. Geodynamics uses data from geology, geophysics, and other sciences and makes extensive use of mathematical and physical modeling. Geodynamics studies the nature of the underlying processes that arise because of the Earth's evolution and cause the movement of matter within the planet. The study of the stress state of the Earth's crust and mantle is one of its main tasks [1]. Stresses are a peculiar characteristic of the tone of the Earth's crust and mantle, which determines the course of geological and geophysical processes. The study of the stress state of the Earth's crust has not only important scientific but also practical importance. The fact that rocks experience great stress has long been well-known. Tunnel builders encountered it as early as the last century. Knowledge of the stressed state of rock massifs can increase the reliability of structures by several times.

The study of the stressed state of the Earth's crust is carried out on the basis of the combined use of various methods. The leading role belongs to geological methods of reconstruction of stress fields, which took place in the past during the development of typical structural elements of the Earth's crust. Since geological investigations alone are not sufficient to determine the distribution of stresses in the Earth's crust, theoretical calculations and modeling of stress fields are of great importance. A possible limitation of the model conclusions is connected with the inaccessibility of direct experiments at Earth depths for the establishment of real values of physical parameters.

There are a sufficient number of publications assessing the stress state from earthquake mechanisms and geological data [2-5]. An extensive review of stresses worldwide is given in [6,7]. Some of this information is based on instrumental data, but most of it is derived from seismological data. Such data are also available for certain areas of Central Asia [8-13]. However, all of them are limited to local reconstruction of the geodynamic type of deformation (tension-shear-compression) and do not give a quantitative characteristic of the stress state of the Central Asian crust as a whole. Moreover, such works often do not take into account the basic provision of mechanics - equations of equilibrium and boundary conditions, which is rightly criticized by Mukhamediev [14,15].

Gzovsky analyzed numerous studies by different authors on tectonic movements during the last 30 million years and on seismicity over 50 years for the territory of the former USSR [16]. He estimated approximate values of possible maximal tangential stresses at depths of 15-20 km, where the sources of strong earthquakes are mostly concentrated. Particularly in the territory of Central Asia, they obtained values in the range

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from 0.1 to 1.5 (10^8 Pa), with the lowest values in the Turan plate and Central Kazakhstan shield and the highest values in the South Tien-Shan and Hindu-Kush parts of the Pamir-Hindukush zone In the Northern Tien Shan, Fergana intermountain trough, on the big part of Alay and on the territory of Pamir maximum tangential stress is estimated 0.7-1 (10^8Pa), on the rest territory 0.4-0.6 (10^8Pa).

To determine stresses according to the continuum equations, boundary conditions have to be set for equilibrium equations. Unfortunately, this is difficult for geodynamic problems. At best, we know the fragmentary stresses extracted in wells. For a block of rock in natural occurrence, such information is usually absent or extremely unreliable since it can be derived only from speculative constructions and assumptions. Obtaining quantitative stress data based on a mathematical model is the main goal of our work.

Historically, Central Asia underwent a pre-platform regime lasting approximately 330 million years from the beginning of the Cambrian to the end of the Permian, then a platform regime lasting approximately 220 million years to the beginning of the Neogene, and finally, an orogenic regime lasting more than 27 million years. According to the predominant opinion of the experts, it is believed that the region's tension is caused by the action on the Turan plate of the compressive forces of the Punjab wedge of Hindustan and the Arabian plate relative to the Eurasian plate. Extrapolation of the modern velocities of relative plate movement on the southern boundaries gives estimates of 25-46 km/mln years.

Methods

Analyzing the orientation and magnitudes of stresses in different belts of the world, Nikolaev et al. concluded that the stress field in the lithosphere is the result of modern forces rather than residual stresses from past tectonic activity [17]. Therefore, we decided to build a model of the stress state of the Central Asian lithosphere based on the current picture of the deformation of its lithosphere. On the basis of elasticity equations, the inverse problem is solved. Atabekov defined the boundary conditions by means of which the received stresses on the territory of Central Asia do not contradict the empirical values described above [18]. According to the properties of stone in laboratory conditions, the elastic modulus of granite is approximately 39-40 MPa, and the end shear modulus is approximately 14-34 MPa [19]. From this follows the question of whether it is possible to apply the methods of the theory of elasticity at such stresses. However, at the depths of the Earth's crust, rock strength increases linearly with increasing pressure (Byerlee's law) [20-23]. Therefore, to calculate the stresses of the Earth's crust, it is quite possible to apply the methods of the theory of elasticity. Our study of the earthquake energy determined on the basis of seismological data within the last 120 years shows that the strongest earthquakes in Central Asia with geographical coordinates (36°-46°E; 56°-76°N) (Figure 1) occur in the interval of 15-20 km in the Earth's crust (Figure 2). The total energy was calculated from the earthquake magnitude using the known formulas LgE =10K and K =1.8 M+4 available for the territory of Central Asia.

Figure 1. Geographic location of the territory of Central Asia for geodynamic modeling.

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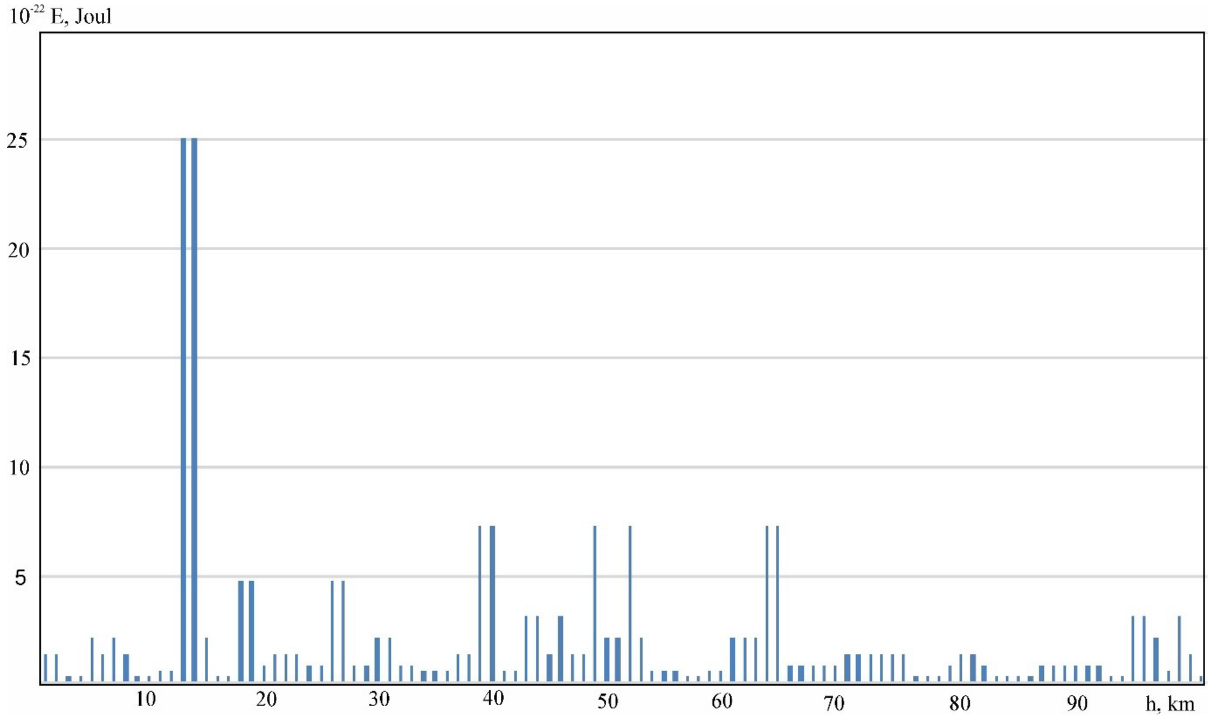

Figure 2. Released total seismic energy E by depth h over the last 120 years in the crust of Central Asia.

The stress state of the lithosphere is determined by momentum equilibrium equations:

$$\sigma_{ij,j} + F_i = 0, \quad (1)$$

$$\mu_{ij,j} + \varepsilon_{ijk}\sigma_{jk} + M_i = 0, \quad (2)$$

i,j=1,2,3

where σ_{ij} and μ_{ij} are the components of the force and moment stress tensor, comma j means differentiation by Cartesian coordinates x_j, F (0,0, ρg) is the mass force, ρ is the density, g is the acceleration of gravity, ε_{ijk} is the Levi – Civita tensor, and Mi is the mass moment having the size of the moment divided by the volume.

The main difficulty in the application of asymmetric theory lies in the difficulties encountered in determining the constants connecting the generalized stresses with the kinematic parameters for obtaining the constitutive relations of materials. There is a limited number of experiments that allow the identification of six elastic Cosserat constants only for the simplest materials that are known. Considering this uncertainty, the rotation can be roughly expressed as before, using the formulas $\omega k = \varepsilon_{ijk} u_{i,j}$, and then $\mu_{ij} = 0$ and formula (2) expresses the asymmetry of the stress tensor corresponding to the moment:

$$\sigma_{ij} = \sigma_{ji} + \varepsilon_{ijk} M_k \quad (3)$$

Equations (1-3) are supplemented with boundary conditions. There are no stresses on the Earth's surface, and on the contact of the lithosphere with the asthenosphere, the normal stresses are equal to the weight of the overlying layers, and the tangential stresses can be assumed to be frictional forces arising from the relative movements of the more mobile asthenosphere with the lithosphere:

$$\sigma_{33}\big|_H = 0,\ \sigma_{13}\big|_H = 0,\ \sigma_{23}\big|_H = 0, \quad (4)$$

$$\sigma_{33}\big|_h = \rho g(h-H),\ \sigma_{13}\big|_h = k_a\sigma_{33}\big|_h, \sigma_{23}\big|_h = k_a\sigma_{33}\big|_h \quad (5)$$

Here, $H=H_{(x1,x2)}$ is the relief of the Earth's surface, $h=h_{(x1,x2)}$ is the lower boundary of the lithosphere, and k_a is the coefficient of friction of the lithosphere with the asthenosphere. The problem was solved relative to the Eurasian plate, i.e., the displacements corresponding to the lateral boundary of the Eurasian plate were considered zero. There are no boundary conditions on the other lateral boundaries of the selected volume. According to tectonic data, the lithosphere of the whole region is divided into several conditionally homogeneous blocks, which differ in physical parameters.

In the Cartesian coordinate system placed on the Earth's surface, the x_1 axis is directed parallel, the x_2 axis is directed along the meridian, and the x_3 axis is directed vertically downwards. Geodynamic features of the problem statement, together with the geometric dimensions in plane and thickness of the lithosphere h, allow simplification of the three-dimensional equations (1) by the following formula:

$$\overline{w}(x_1, x_2) = \frac{1}{[h - H(x_1, x_2)]}\int_H^h w(x_1, x_2, x_3)dx_3, \quad (6)$$

where the dash means averaging over x_3. In geodynamics, the thickness of the lithosphere h is usually assumed to be constant and equal to 100 km (1). This method reduces the solutions of the problem to a two-dimensional one but with the possibility of preserving some three-dimensional specificity of the solutions. As a result, the following Lame equation was obtained for the averaged elastic displacements u_1 and u_2:

$$\Delta\overline{U} + \frac{1}{(1-2\nu)} graddiv\overline{U} = \overline{F} \quad (7)$$

$$F_1 = -\overline{\frac{\partial M_2}{\partial x_3}}\frac{p}{(1-H/h)}\overline{\sigma_{11}}\frac{\partial H}{\partial x_1} + \frac{p}{(1-H/h)}\overline{\sigma_{12}}\frac{\partial H}{\partial x_2} - \frac{\partial}{\partial x_1}\{\frac{p}{(1-H/h)}[(\frac{2(1-\nu)}{(1-2\nu)}\frac{\partial H}{\partial x_1}\overline{u_1} + \frac{2\nu}{(1-2\nu)}\frac{\partial H}{\partial x_2}\overline{u_2})]\} \quad (8)$$
$$\frac{\partial}{\partial x_2}\{\frac{p}{(1-H/h)}\frac{G}{G_0}[\frac{\partial H}{\partial x_2}\overline{u_1} + \frac{\partial H}{\partial x_1}\overline{u_2})]\} - \frac{\nu}{(1-\nu)}\frac{\partial(u_3^h - u_3^H)}{\partial x_1} - -\frac{k_a\rho g(h-H)}{2G_0},$$

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$$F_2=-\frac{\overline{\partial M_3}}{\partial x_1}-\frac{\overline{\partial M_1}}{\partial x_3}\frac{p}{(1-H/h)}\overline{\sigma_{12}}\frac{\partial H}{\partial x_1}+\frac{p}{(1-H/h)}\overline{\sigma_{22}}\frac{\partial H}{\partial x_2}-\frac{\partial}{\partial x_1}\{\frac{p}{(1-H/h)}\frac{G}{G_0}[\frac{\partial H}{\partial x_2}\overline{u_1}+\frac{\partial H}{\partial x_1}\overline{u_2})]\}-$$
$$\frac{\partial}{\partial x_2}\{\frac{p}{(1-H/h)}[(\frac{2\nu}{(1-2\nu)}\frac{\partial H}{\partial x_1}\overline{u_1}+\frac{2(1-\nu)}{(1-2\nu)}\frac{\partial H}{\partial x_2}\overline{u_2})]\}-\frac{\nu}{(1-\nu)}\frac{\partial(u_3^h-u_3^H)}{\partial x_2}--\frac{k_a\rho g(h-H)}{2G_0}, \quad (9)$$

$$\overline{\sigma}_{33}(x_1,x_2)=\frac{\rho g(h-H)}{2G_0}=\frac{2\nu}{(1-2\nu)}(\frac{\partial\overline{u}_1}{\partial x_1}-\frac{p\overline{u_1}}{(1-H/h)}\frac{\partial H}{\partial x_1}+\frac{\partial\overline{u}_2}{\partial x_2}-\frac{p\overline{u_2}}{(1-H/h)}\frac{\partial H}{\partial x_2})$$
$$+\frac{2(1-\nu)}{(1-2\nu)}\frac{(u_3{}^h-u_3{}^H)}{(1-H/h)} \quad (10)$$

$$u_3(x_1,x_2,H)-u_3(x_1,x_2,h)=(1-H/h)\frac{\nu}{2(1-\nu)}(\frac{\partial\overline{u}_1}{\partial x_1}-\frac{p\overline{u_1}}{(1-H/h)}\frac{\partial H}{\partial x_1}$$
$$+\frac{\partial\overline{u}_2}{\partial x_2}-\frac{p\overline{u_2}}{(1-H/h)}\frac{\partial H}{\partial x_2})+\frac{(1-2\nu)}{4(1-\nu)}\frac{\rho g(h-H)}{G_0} \quad (11)$$

Here, Ū-two-dimensional vector with components of averaged horizontal displacements, Δ - two-dimensional Laplace operator, ν - Poisson's ratio, u_3^h, u_3^H vertical displacements at the base of the asthenosphere and on the surface of the earth. The p parameter appears when applying formula (6) to the shear stress on the free surface of the earth and expresses the ratio of stresses on the surface and the expected average depth. The derivatives M_i are averaged by formula (6).

In the equations, linear variables are scaled with respect to *h*, and stresses and moments are scaled with respect to the average elastic shear modulus G_0.

In the geographical system of coordinates, M_i is expressed using the parameters of the failure plane angles of strike ϕ, slope λ, dip δ, and moment M_0. According to Landau and Lifshitz [24], moment M_0 is equal:

$$M_0=\mu AU \quad (12)$$

where μ is the rigidity in the source region, and A is the area over which the shear dislocation U has been averaged. The moments M_i and M_0 are expressed through the parameters of the failure plane according to Aki by the following formulas [25]:

$$M_1=-M_0(\cos\delta\cos\lambda\sin\varphi-\cos2\delta\sin\lambda\cos\varphi),$$
$$M_2=-M_0(\cos\delta\cos\lambda\cos\varphi+\cos2\delta\sin\lambda\sin\varphi),$$
$$M_3=M_0(\sin\delta\cos\lambda\cos2\varphi+1/2\sin2\delta\sin\lambda\sin2\varphi) \quad (13)$$

For example, in the special case of a double dipole at the location (x10, x20, x30) of the nodal plane by the azimuth φ=0°, slope δ=90° and slip angle λ =90°, they are the next:

$$M_1=M_3=0$$
$$M_2=\frac{\Delta\sigma}{G_0}\left(\frac{r_0}{\sqrt{\sum_{i=1}^{3}(x_i-x_{i0})^2}}\right)^{n_{eff}} \quad (14)$$

Here, M_3 is expressed as an energetic model according to Riznichenko [26], where r_0 is the relative radius of the reference sphere, n_{eff} is the divergence coefficient, and $\Delta\sigma$ is the stress relieved.

The solution to the problem with incomplete boundary conditions is naturally not unique. Additional information is required to construct a solution to such problems. For instance, in the case of a single equation domain (*i*=1), the solution can be represented as a sum with coefficients of elasticity problem eigenvalues and by selection of coefficients by the least squares method involving some a priori information [27,28]. In some cases, the solution to Fredholm equations with respect to the boundary values, and then additional information is used. We do not question the existence of the set problem and acceptable solution constructed based on concrete geodynamic conditions. According to the hypothesis described above, the stress-strain state of the considered territory is caused by the compression of the Eurasian plate on one side and the Indian and Arabian plates on the other side. It would be naive to look for the solution of a heavy elastic prism resting on a fixed base, which receives the vertical displacements corresponding to the real relief of the earth under the action of lateral compressions. At least in the formulation of small deformations, this is nonsense because the height of the real terrain reaches 6-7 km. Therefore, by numerical experiment, Atabekov [18] decided to construct the boundary conditions in such a way that the obtained solution under these boundary conditions approximately coincided with the established empirical values in Gzovsky [16]. At the beginning of the experiment, the average σ_{ij} in the right part of formulas [17,18] was used to solve the plane problem, the boundary conditions for which were selected preliminarily according to hypothetical data about the velocities of the Indian and Arabian plates relative to the Eurasian plate. Namely, the stress relations created by the Indian and Arabian plates at the southern corners of the Turan plate were taken equal to their velocity relations with uniform interpolation. Each stress on the right and left sides of the rectangle, which limits our region in plan, was interpolated to the upper boundary of the Eurasian plate, which is assumed stationary. Figure 3 shows the corresponding boundary conditions for this problem and the obtained stress intensity isolines σ_i, determined by the formula:

$$\sigma_i=\frac{\sqrt{2}}{2}\sqrt{(\sigma_{11}-\sigma_{22})^2+\sigma_{11}{}^2+\sigma_{22}{}^2+6\sigma_{12}{}^2} \quad (15)$$

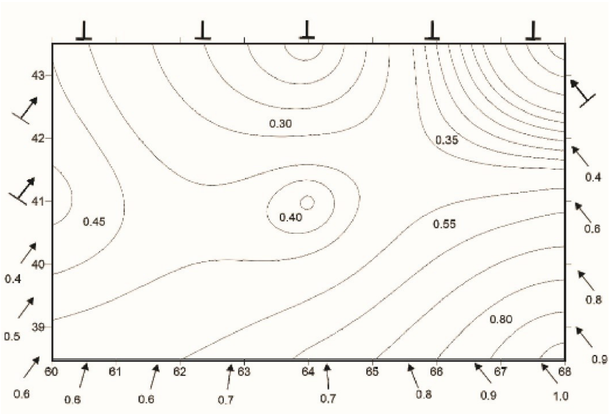

Figure 3. Isolines of stress intensity σ_i (in 10^2 MPa) from the solution of the generalized planar stress state problem, with boundary conditions shown in the figure. The arrows indicate the directions of the forces, the inverse T arrow means no displacement, and the T with arrows means a slippery boundary.

The reconstructed stresses of an elastic problem are used as initial data for other problems. For example, to solve the problem of the modern movements of the Earth's crust in territories of Central Asia by Stokes equations Atabekov [29]. The tectonic flow of mountain masses occurring in a relatively short time can provide important information. One of the important parameters in seismic zoning is the displacement rate gradient. In contrast to the tectonic stresses, which change weakly, the seismotectonic flow of rock masses occurring in a

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relatively short time can provide important information for tectonic zoning. We apply to the Stokes equations similar procedures described above for the elastic problem. By relating the averaged displacement velocities h/t_0, and stresses to μ_0/t_0 (μ_0-the average viscosity of the constituent regions, t_0-the time scale), we obtain the following dimensionless Stokes equations:

$$-grad\,\overline{p}+\frac{\mu}{\mu_0}\Delta\overline{v}=\overline{F}, \tag{16}$$

$$F_1=-\frac{\overline{\partial M_2}}{\partial x_3}\frac{p}{(1-H/h)}\overline{\sigma_{11}}\frac{\partial H}{\partial x_1}+\frac{p}{(1-H/h)}\overline{\sigma_{12}}\frac{\partial H}{\partial x_2}-\frac{\partial}{\partial x_1}(\frac{p}{(1-H/h)}\frac{\partial H}{\partial x_1}\overline{v_1})- \tag{17}$$
$$\frac{\partial}{\partial x_2}[\frac{2p}{(1-H/h)}\frac{\mu}{\mu_0}(\frac{\partial H}{\partial x_2}\overline{v_1}+\frac{\partial H}{\partial x_1}\overline{v_2})]-\frac{1}{(1-H/h)}[v_3(x_1,x_2,H)\frac{\partial H}{\partial x_1}+\frac{\partial v_3(x_1,x_2,H)}{\partial x_1})],$$

$$F_2=-\frac{\overline{\partial M_3}}{\partial x_1}-\frac{\overline{\partial M_1}}{\partial x_3}\frac{1}{2(1-H/h)}\overline{\sigma_{21}}\frac{\partial H}{\partial x_1}+\frac{1}{2(1-H/h)}\overline{\sigma_{22}}\frac{\partial H}{\partial x_2}-\frac{\partial}{\partial x_1}[\frac{2p}{(1-H/h)}\frac{\mu}{\mu_0}(\frac{\partial H}{\partial x_2}\overline{v_1}+\frac{\partial H}{\partial x_1}\overline{v_2})]-$$
$$\frac{\partial}{\partial x_2}(\frac{p}{(1-H/h)}\frac{\partial H}{\partial x_2}\overline{v_2})-\frac{1}{(1-H/h)}[v_3(x_1,x_2,H)\frac{\partial H}{\partial x_2}+\frac{\partial v_3(x_1,x_2,H)}{\partial x_2})] \tag{18}$$

$$v_3(x_1,x_2,H)=(1-H/h)\frac{\partial\overline{v_1}}{\partial x_1}+\frac{\partial\overline{v_2}}{\partial x_2}-p(\frac{\partial H}{\partial x_1}\overline{v_1}+\frac{\partial H}{\partial x_2}\overline{v_2}) \tag{19}$$

Formula (19) is obtained by averaging the continuity equation. In this case, the values of the vertical velocity on the lithosphere are taken equal to zero $v3(x_1, x_2, h)=0$.

The time scale is chosen from the equality of dimensionless tangential stresses in the Lamé and Stokes equations:

$$\frac{\overline{\sigma_{ij}}^{visc}}{\frac{\mu_0}{t_0}}=\frac{\overline{\sigma_{ij}}^{elast}}{G_0} \tag{20}$$

The boundary conditions for creep motion were chosen according to the modern velocities of the tectonic plates and stresses and the found stresses for interior points of the Lame equation. Equations (7) and (14) were solved by iteration. As a zero approximation, and u3h=0. In each iteration, the system was solved by the BEM. The integral equations of the method have a standard form [30]:

$$c_{ij}(x)w_j(x)+\int_S p^*{}_{ij}(x,\xi)w_j(\xi)dS_\xi=\int_S w^*{}_{ij}(x,\xi)p_j(\xi)dS_\xi+\int_\Omega w^*{}_{ij}(x,\xi)b_j(\xi)d\Omega \tag{21}$$

Here, $S=\bigcup_{i=1}^{n}S_i$, $\Omega=\bigcap_{i=1}^{19}\Omega_i$, S_i are the boundaries of the two-dimensional domain Ω_i, $r_j=x_j-\xi_j$, b_j are the right-hand sides of the equations, and p_{ij}^*, w_{ij}^* are fundamental solutions. The coefficient $c_{ij}(x)$ expresses the regularity of the boundary curve and the choice of the fundamental solution. For a regular curve, it is equal to π. In the program, it is calculated automatically based on the approximated broken line of the boundary curve and is equal to the internal angle of the boundary at point x. For the Lamé equations (w=ū), they have the following form (31):

$$u^*{}_{ij}=\frac{1}{8\pi G(1-\nu)}\left\{(3-4\nu)\ln\frac{1}{r}\delta_{ij}+r_{,i}r_{,j}\right\} \tag{22}$$

$$p^*{}_{ij}=-\frac{1}{4\pi(1-\nu)}\left\{\frac{\partial r}{\partial n}\left[(1-2\nu)\delta_{ij}+2r_{,i}r_{,j}\right]-(1-2\nu)(r_{,i}n_{,j}-r_{,j}n_{,i})\right\} \tag{23}$$

$<i,j=1,2>$.

For the Stokes equations ($w=\overline{v}$) Ladyzhenskaya [31]:

$$u_{ij}^*=-\frac{1}{4\pi\mu/\mu_0}\left[\delta_{ij}\ln r-r_{,i}r_{,j}\right] \tag{24}$$

$$p_{ij}^*=-\frac{r_{,i}r_{,j}}{\pi r}\frac{\partial r}{\partial n} \tag{25}$$

The averaged stresses in the case of creep flow are constructed by the formula:

$$\overline{\sigma}_{ij}=-\delta_{ij}p+\frac{\mu}{\mu_0}(\frac{\partial\overline{v}_i}{\partial x_j}+\frac{\partial\overline{v}_j}{\partial x_i}) \tag{26}$$

Here's

$$\overline{p}(x)=\int_\Omega q^k(x,\xi)b_k(\xi)d\Omega_\xi-\int_S q^k(x,\xi)\overline{\sigma}_{kj}n_jdS_\xi-2\frac{\mu}{\mu_0}\int_S\frac{\partial q^k}{\partial x_j}\overline{v}_kn_jdS_\xi \tag{27}$$

$$q^k(x,\xi)=\frac{1}{2\pi}\frac{\partial}{\partial x_k}\ln\frac{1}{r(x,\xi)} \tag{28}$$

Integral equations (21) contain both known and unknown boundary values. In numerical implementation, the calculated integrals with known boundary values of velocities and stresses are separated into matrix *B*, which constitutes the right-hand side of the algebraic equation. The integrals with unknown displacements and stresses constitute matrix *A*. The integrals over S are separated into the sum of integrals S_i with different physical parameters, taking into account the conjugation condition of stresses and velocities. The boundaries S_i are divided into linear elements, Ω_i into triangles, the vertices of which are interior and boundary points.

Results

The search for the solution of the main problem is reduced to the solution of a number of direct problems by varying the boundary stresses. In the initial stage, the boundary conditions were taken as for the plane problem. The stresses obtained from the solution of the previous cycle were on the right side of (7-9) in the subsequent cycle. Using the formula (11), the corresponding modified relief was constructed as u_3 (x_1,x_2, H). Figure 4 shows some variants of the relief obtained in the numerical experiment. The first picture built on the topological map of Central Asia was adopted for the initial stage of the numerical experiment. In each cycle, the boundary conditions were varied so as not to spoil the real relief and to obtain stresses close to the empirical data Gzovsky [16]. This was the essence of the numerical experiment.

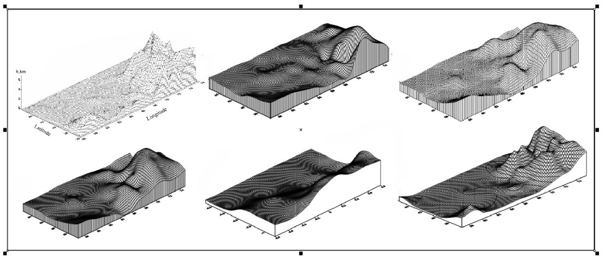

Figure 4. Different variants of the relief obtained during the numerical experiment. The first picture is a modern relief constructed according to a topographic map; the last picture is constructed according to the solution of the final cycle of the numerical experiment. On the horizontal - east longitude and north latitude in degrees.

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Currently, not the entire territory of Central Asia is covered by a network of GPS data that serve to monitor the movement of the Earth's crust to analyse and assess the stress state of the geological environment and forecast changes in the subsurface under the influence of natural and anthropogenic factors. However, unlike horizontal velocities for which there are fixed objects taken as reference points, monitoring vertical velocities is difficult. As a result of solving the Stokes problem, we can construct by formulas (16-20) a field of vertical displacement velocities, which are not instrumentally available everywhere. Therefore, the creation of a numerical model of the stress state of the Central Asian territory serves as an invaluable contribution of mechanics to geodetic surveys.

One of the seismically active territories of Uzbekistan is the Ferghana depression (marked with a triangle in Figure 1), bounded by active Talas-Ferghana, Aksu-Maydantal-Bogonalin, and Gisar-Kokshal faults, containing North Ferghana and South Ferghana deep faults, which determine the main tectonic weather in this region. Within its limits, there is a fairly large reserve of hydrocarbons. During the historical period, strong earthquakes with magnitudes of M> 7 occurred here. Its main tectonic feature is that, under near-meridian compression, the depression has a rift character, characteristic of extension zones. Such features of the geodynamics of the region require further clarification. For modeling, the territory was divided into somewhat conditionally homogeneous blocks, as the boundaries of which we took deep faults. As a result of the calculation by formulas (16-20, 24-28), the field of stresses and velocities of the region was constructed. Figure 5 shows the averaged horizontal shear stresses at depths of 15 km in this area. They mark the most vulnerable places where there may be dangerous earthquakes in the future. Figure 6 shows the vertical movement velocity calculated by formula (19). They complement the available GPS data and, together with stresses, are valuable tectonic material for seismic hazard prediction.

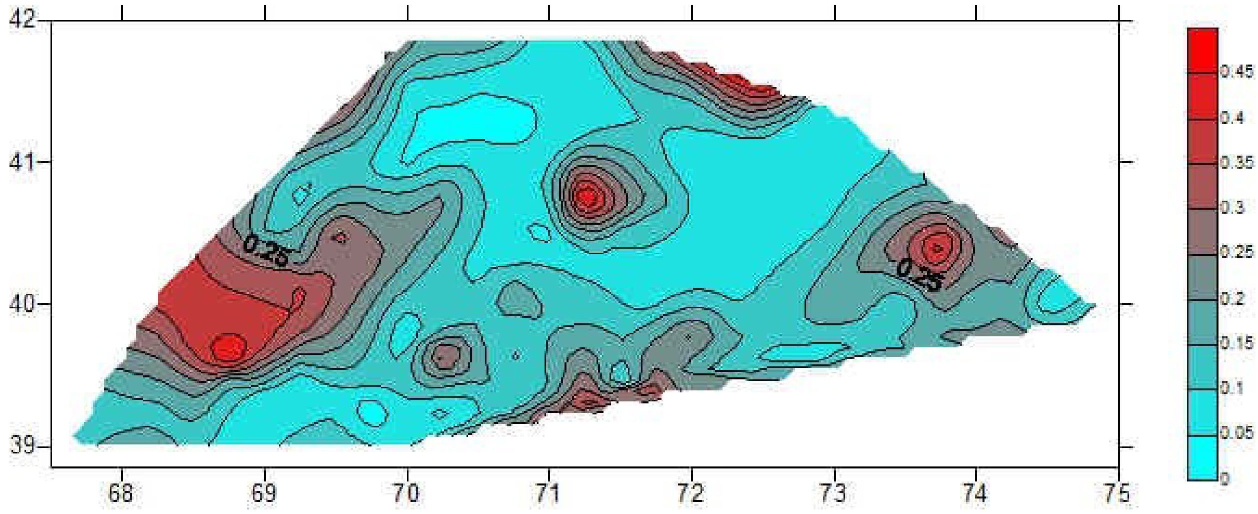

Figure 5. Shear stresses (10^8 Pa) in the Earth's crust of the Feghana Depression at the level of 15-16 km. On vertical North latitude, on horizontal East longitude in degrees.

Figure 6. The velocity of vertical displacement (cm/year) in the territory of the Ferghana Depression. East longitude and north latitude in degrees.

The formulas (7-11, 21-23) make it possible, on average, to estimate changes in the stress state of the Earth's crust due to earthquakes. Using simplified models of the earthquake focus mechanism $\varphi=0°$, $\delta=90°$, $\lambda=90°$, we can estimate how the shear stresses of the Earth's crust in Central Asia approximately change during specific earthquakes. For the example of a recent earthquake that occurred in the territory of Tajikistan (38.07^{0}N|73.20^{0}E shown as a polygon in Figure 1) on 23.02.2023, 00:37:19 GMT, with a magnitude of M0=6.8 and a focal depth of h=21 km, the stresses were calculated (Figure 7). In this case, this applies because this earthquake occurred along an active fault that is located subparallel. As a variation, the stress difference obtained by equations (8-9) at zero and nonzero values of the moments were taken.

Thus, the proposed methods can be applied to find the stress-strain state of other regions, taking into account their peculiarities.

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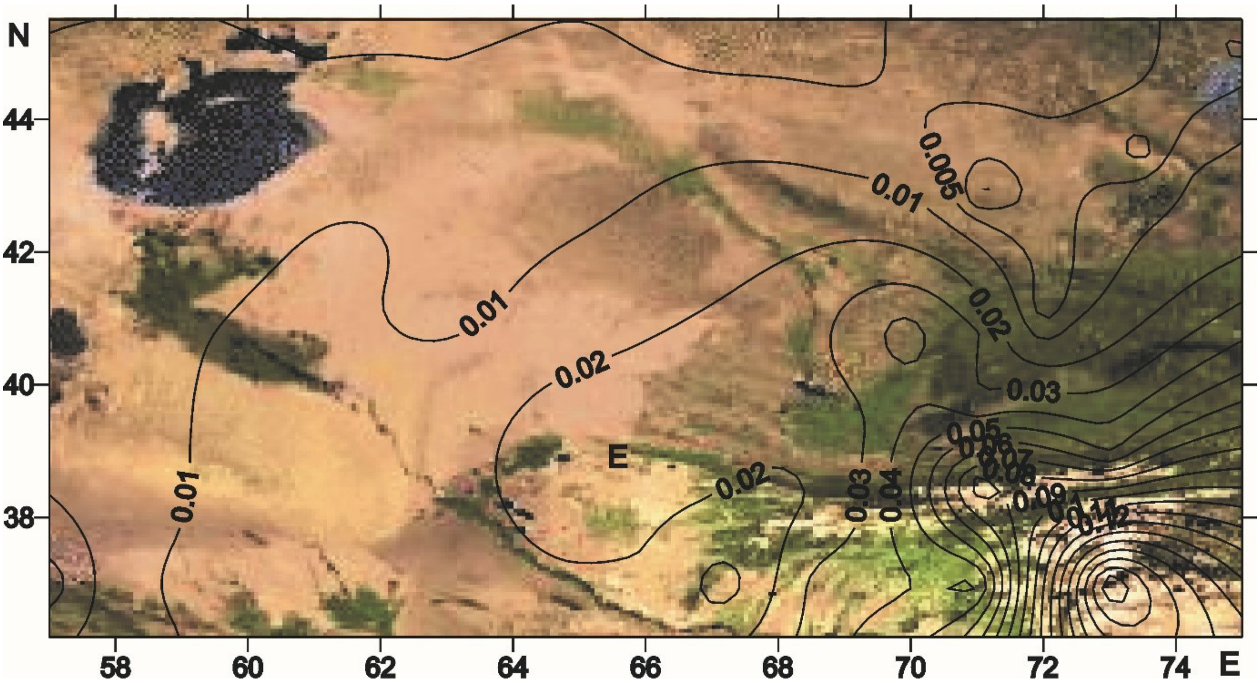

Figure 7. Relative variation in shear stresses (in %) in the Earth's crust of Central Asia due to earthquakes (23.02.2023, 00:37:19 GMT, M=6.8, h=21 km) in Tadjikistan. On vertical North latitude, on horizontal East longitude in degrees.

Conclusions

Based on the continuum equations, a model of the stress state of the Central Asian lithosphere has been created. Given the peculiarities of geodynamic problems, the three-dimensional Lame and Stokes equations are averaged in-depth, taking into account the topography of the region. Numerical solutions were obtained using the method of boundary integral equations for zonally homogeneous bodies. The stresses obtained by the solution of the Lamé problem are used to reconstruct the modern movements of the Earth's crust in the local territory of Central Asia using the Stokes equations. Stresses and horizontal and vertical displacement velocities serve as additional information for monitoring the Earth's interior of Central Asia.

Disclosure statement

No potential conflict of interest was reported by the author.

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